Narayan Das

A Model to Study Changes in Current Fertility Under Different Sex Preferences

Introduction

AVAILABLE research evidence leaves no doubt that, at least in some societies and under certain conditions, the sex of a child is important to parents. In certain societies, sex preference has acquired greater significance for fertility behaviour because family size norms have gone down and contraceptive use has gone up (Das, 1986). If the sex of their children matters to parents, it is interesting to understand the implications of allowing them to achieve their desired sex composition for the overall level of fertility. In this regard, various probablity models have been developed, showing in general that if the sex bias and fertility are stable, the expected family size will increase with increasing preference for one sex over the other. Couples wishing to have at least one son, or at least one son and one daughter, keep having children until they satisfy this desire of theirs and so their family size would be higher than it would otherwise be. The implications of allowing couples to attain the desired family size composition at the current fertility levels are, however, not clearly known. We assess them here by developing applied probability models. It is needless to emphasize that this information has important policy implications. The only study which throws some light in this regard is that of Talwar(1975). By assuming the sex of a live birth to be a binomial event, he attempts to assess the contribution of desires for specific sex composition to the level of birth rate in a population. The birth order distribution of annual births in India is utilized in order to quantify the contribution to the birth rate of various desired compositions in a family, and the level of birth rate if specific reproductive patterns are adopted. The model is however not flexible enough to study its effect on some refined current fertility indices.

The purpose of this research is, therefore, to develop a mathematical model which may help answer the related queries at the macro level without involving much mathematical intricacy. For illustration, Indian data have been considered.

Model

The model to study the effects of allowing couples to attain specified family size composition on fertility, can be looked upon as a controlled experiment. Mainly two sets of current fertility rates would be obtained, one assuming usual reproductive behaviour (where reproduction is, by and large, at the observed level, i e. unaffected by any specific planning-control set) and other with specific rules for stopping after achieving certain specified family size composition (experimental set). All the input parameters except those of stopping rules will be identical for deriving these two sets of fertility rates. The differential in the fertility rate of the two sets (control and experimental) is a measure of the impact of stopping rule adopted by the parents regarding the sex composition of their children, on fertility.

To obtain fertility rate, the analysis is done in two segments. The first segment derives estimates of birth probabilities for a given age and age at marriage of a woman through probability model. The second segment involves estimation of various fertility rates from the age at marriage and age specific birth probability matrices derived in the first segment. This is done through a simple projection technique (Venkatacharya, 1972).

The underlying assumptions of the model to derive birth probabilities specific to age and age at marriage are as follows :

1. A woman is not pregnant and is fecund at the time of consummation of her marriage and continues to be in marital union until she attains 45 years of age.
2. Let h be the infecundable period associated with a conception. We assume, for the sake of simplicity, that there is a one to one correspondence between a conception and a live birth. We also assume that the length of this infecundable exposure is constant for all the ages of women.
3. The probability that $(i + 1)$th conception occurs during the time interval $(t, t + dt)$ given that the ith conception occurred prior to $t - h$, is $\pi dt + 0(dt.)$, where $t > ih$, $\pi > 0$ and zero otherwise. This assumption is equivalent to the assumption that waiting time for a conception after a woman becomes fecund following a live birth or after her marriage in case of first conception follows exponential distribution with mean $(1/\pi)$.
4. The probability that a fecund woman is exposed to the risk of conception at age x is β and the probability that she is not exposed is $1 - \beta$

5. The probability that a child born will be male is p. The probability of twins or multiple births is zero. The probability that a child will be a boy is assumed to be constant, and same for all parents.
6. It is assumed that all women get married by 35 years of age.

Some of the assumptions are no doubt strong but they may be considered as first approximations to reality process. In the following section only the main steps underlying the model are given; the details are given in Appendices I and II.

Derivation of Birth Probabilities

Let E_k $(S(b, g, s))$ be the event that a birth takes place to a woman in the kth year of marriage in the experimental group where a couple is assumed to adopt scheme $S(b, g, s)$ of preference for sex and/or size. As per scheme $S(b, g, s)$ a couple will stop reproduction as soon as they have b sons and g daughters or a total of s children $(s \geqslant b + g \geqslant 1, b, g \geqslant 0)$. In order to derive the probability of giving birth in the kth year, $P(E_k(S(b, g, S))) = P(E_k)$, it is first necessary to understand the chances of not achieving desired sex composition in the $(k - 1)$th year or before.

The probability that a couple will not achieve the sex composition of b sons and g daughters at parity m or before, denoted by $Q_m(b, g)$, is given by

$$Q_m(b, g) = \begin{cases} F_1(m, p, b) + F_2(m, p, g) & \text{if } m \geqslant b + g \\ 1 & \text{if } m < b + g \end{cases}$$

where

$$F_1(m, p, b) = \begin{cases} \sum_{0}^{b-1} \binom{m}{r} p^r q^{m-r} & \text{if } b \geqslant 1 \\ 0 & \text{if } b = 0 \end{cases}$$

and

$$F_2(m, p, g) = \begin{cases} \sum_{0}^{g-1} \binom{m}{r} q^r p^{m-r} & \text{if } g \geqslant 1 \\ 0 & \text{if } g = 0 \end{cases}$$

It is easily seen that $F_1(m, p, b)$ $(F_2(m, p, g))$ represents the probability of having less than b sons (g daughters) by the mth parity (for details of the proof see Appendix I). Under the condition that all fecund women are exposed to the risk of conception and the couples adhere to stopping rule $S(b, g, s)$, the probabilities of a woman giving birth in the 1st, and kth year of marriage (for $k \geqslant 2$) are given by

$$P^*(E_1) = 1 - e^{-\pi t_1}$$

and for $k \geqslant 2$

$$p^*(E_k) = e^{-\pi t} \; k-1 \; (1 - e^{-\pi}) + \sum_{m=1}^{\min\{k-1,\, s-1\}} Pr \cdot [r_1 < Z_{m+1} < r_1 + 1] \cdot Q_m(b, g)$$

where,

$$Pr \cdot [r_1 < Z_{m+1} < r_1 + 1] = \begin{cases} \sum_{n=0}^{m} \frac{e^{-\pi r_1} (\pi r_1)^n}{n!} - \sum_{n=0}^{m} \frac{e^{-\pi(r_1+1)} \{\pi(r_1+1)\}^n}{n!} & \text{if } r_1 > 0 \\ 1 - \sum_{n=0}^{m} \frac{e^{-\pi(r_1+1)} \{\pi(r_1+1)\}^n}{n!} & \text{if } r_1 \leqslant 0 < r_1 + 1 \\ 0 & \text{if } r_1 + 1 \leqslant 0 \end{cases}$$

with, $Z_{m+1} + W_1 + W_2 + \ldots + W_{m+1}$, $r_1 = t_{k-1} - mh$, and $t_k = k - 3/4$.

***Remarks*:**

(1) It may be noted that 's' is introduced under the summation to put an upper limit on the total number of children (live births),

(2) If a couple has a particular sex preference, say *b* sons and *g* daughters but no upper limit on the total number of children, the above formulae can still be used by taking *s* large (say 50).

(3) When a couple has no particular sex preference and wishes to stop reproduction with s children, the above formulae can be used with the modification that

Qm (b, g) = 1 for all *m*.

The birth probabilities derived above are in fact conditional probabilities, the condition being that a woman is not sterile. The unconditional birth probability,fex,y for a woman of age *x* who married at age *y* is given by

$$P(E_k) = f^*_{x,y} = P^*(E_{x-y+1})\, \beta_x$$

where, β_x is the probability that a woman is not sterile at age x and before;

$x = 15, 16, \ldots, 44;$

$y = 15, 16, \ldots, 35;$ and

$x > y$

Let C_k be the event that a birth takes place to a woman in the kth year of marriage in the control group where a couple is assumed to follow no stopping rule regarding adoption of any particular sex composition and/or size of the family.

The corresponding derivation of $P^*(C_k)$ (and $P(C_k)$) in the control group is similar to that of $P^*(E_k)$ (and $P(E_k)$) in the experimental group. The only difference is that $Q_m(b, g)$ is not required to be considered in the control group. However, the formula for $P^*(C_k)$ can be derived from the results of experimental group where b and g can be taken very large so that $Q_m(b, g) = 1$ for all m, and s is also taken large enough to remove its effect on the summation in the formulae for $P^*(E_k)$. This is because it is assumed that there is no limit on b, g or s in the control group. Thus one can obtain the formulae for $P^*(C_k)$ and $P(C_k)$ from $P^*(E_k)$ and $P(E_k)$ by taking $Q_m(b, g) = 1$ for all m, with s assumed large. These probabilities can be rewritten as

$$P^*(C_k) = \sum_{m=1}^{k} Pr \cdot [r_1' < Z_m < r_1' + 1]$$

where,

$$Pr \cdot [r_1' < Z_m < r_1' + 1] = \begin{cases} \sum_{n=0}^{m-1} \frac{e^{-\pi r_1'} (\pi r_1')^n}{n!} - \sum_{n=0}^{m-1} \frac{e^{-\pi(r_1'+1)} \{\pi(r_1' + 1)\}^n}{n!} & \text{if } r_1' > 0 \\ 1 - \sum_{n=0}^{m-1} \frac{e^{-\pi(r_1'+1)} \{\pi(r_1' + 1)\}^n}{n!} & \text{if } r_1 \leqslant 0 < r_1'+1 \\ 0 & \text{if } r_1' + 1 \leqslant 0 \end{cases}$$

In the above expressions $r_1' = t_{k-1} - (m - 1)\, h$; $t_k = k - 3/4$ and $Z_m = W_1 + W_2 + \ldots + W_m$, where $W_1, W_2 \ldots W_m$ are independent identically distributed exponential random variables.

Further, the unconditional birth probability, $f^c_{x,y}$ for a woman of age x who married at age y in the control group is given by

$$P(C_n) = f^c_{x,y} = P^*(C_{x-y+1})\, \beta_x$$

where

$x = 15, 16, \ldots, 44;$
$y = 15, 16, \ldots, 35;$ and
$x > y$

Derivation of Current Fertility Rates

Having obtained the estimates of age at marriage and age specific birth probabilities under control and various sex preference assumptions, the corresponding fertility rates and their trends in the next fifteen years are derived from them. For this we need the currently married women by age at each future year. This is obtained by projecting the single year currently married women in 1981 into future years, by making use of appropriate joint survival ratio and taking into account new entrants through marriage at each year. The details of obtaining currently married women by age in a given year J, W_{jx} are shown in Appendix II. It is now possible to derive various measures of current fertility.

Given the constancy of birth probabilities $f_{ex\,y}$ or $f_{cx\,y}$ over the time as well as of the corresponding probabilities (m_v) of marriage (Appendix II, assumption 5), the single year age specific marital fertility rate (Fx is not supposed to show variation over the years and is therefore independent of J. So is the total marital fertility rate (T) However, the ASMFRs by five years age groups, the GMFR (G_J) or crude birth rate (derived later in the section) may be expected to vary, depending on the extent of changes in the population age composition over time. The single year ASMFRs (F_x) as well as TMFR (T), derived from F_x, are independent of J and can be obtained by

$$F_x = P(\text{A married woman of age } x \text{ gives birth to a child})$$

$$= \sum_{y=15}^{35} P(\text{A woman of age } x \text{ gives birth to a child and the woman was married at age } y)$$

$$= \sum_{y=15}^{\min(x,\,35)} f_{x,\,y},\, m_y \qquad (x = 15, 16, \ldots, 44)$$

and

$$T = \sum_{x=15}^{44} F_x$$

where m_y is the probability of a single woman marrying at age y\

$f_{x,\,y}$ = probability of occurrence of a birth to a woman given that she is aged x and is married at age y.

Note that $f_{x,\,y}$ is either $f_{cx\,y}$ or $f_{ex\,y}$ depending on the group (control or experimental) for which the fertility rates are to be obtained.

It is now possible to derive various other measures of current fertility. The absolute number of births to the currently married women aged x in the jth year can be obtained as

$$b_x^J = W_x^J\, F_x \qquad (J = 1981, 1982 \ldots 1996)$$

and the absolute number of births to the women in the age group 15-44 in the

Jth year can be obtained as

$$b^J = \sum_{x=15}^{44} b_x^J = \sum_{x=15}^{44} W_x^J F_x$$

where,

W_x^J = Number of currently married women aged x in the year J.

Relating these births to currently married women, general marital fertility rate (G^J) as well as ASMFRs in the conventional five year age groups (${}_5F_x^J$) for the control and experimental set in a given year J can be obtained :

$$G^J = \frac{b^J}{W^J} = \sum_{x=15}^{44} b_x^J \Big/ \sum_{x=15}^{44} W_x^J, \; (J = 1981, 1982 \ldots 1996)$$

$${}_5F_x^J = {}_5b_x^J \,/\, {}_5W_x^J, \qquad (x = 15, 20, 25, 30, 35, 40)$$

where,

$${}_5b_x^J = \sum_{i=0}^{4} b_{x+i}^J$$

and

$${}_5W_x^J = \sum_{i=0}^{4} W_{x+i}^J$$

Remarks: The above results can also be used to estimate Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR). GRR is a slight modification of the total fertility rate. The only distinction is that the numerator of the GRR is based on female births instead of total births. The NRR also uses the age specific female birth rates; however, it is based on the survivors of a cohort rather than on a cohort without mortality (the cohort is taken directly from a Life Table).

It is also possible to estimate the birth rate of the population under the different sex preference and stopping rule assumptions for the Jth year. To do this the mid-year population for the Jth year is required as the absolute number of births in Jth year is already known. In other words, the mid-year population for each of the years 1981, 1982 . . . 1995 is required. This is done by simply projecting the 1981 census population by age and sex into future years, by making use of appropriate survival ratios which are again selected with the help of the assumed level of male and female e_0^0 of the respective period (see Table 2, Appendix–II). Further, in future years, the births which take place during a year under a particular fertility assumption, are to be added to survivors of the previous year's population. For this we make use of" the absolute number of births in each calender year obtained through the present birth probability matrices and they are distributed by sex assuming the sex ratio at birth to be 105 males per 100 females. The enumerated population of 1981 census is taken to be the population at the beginning of 1981. Therefore, after knowing the population at the beginning of each year during 1981-1996, the midyear population during each of the_years 1981-95 can be obtained and hence the birth rate. Denoting by P_T and $\overline{P_T}$ the population at the beginning and the middle of the calender year T, we have

$$\overline{P_T} = (P_T + P_{T+1})/2 \ (T = 1981, 1982, \ldots, 1995)$$

Then the birth rate can be obtained as

$$BT = b_T/\overline{P_T}$$

where b_T is the number of births in the calender year T.

The entire work has been programmed for computer analysis. As mentioned earlier, data from India are utilized to illustrate the model.

Results

Apart from the control set, 12 hypothetical cases under the experimental set, giving the rules when a couple would stop, have been considered and in each case the expected level of future fertility for attaining the desired family size composition, has been computed for various combinations of fecundability (") and rest period *(h).* It may, however, be noted that these hypothetical cases, especially Rules 1 to 9, are essentially framed on the basis of empirical data to make them more realistic for India. In an All-India Survey carried out by ORG, Baroda, during 1980, it was found that most frequently preferred combination of children is two sons and one daughter (34 percent), followed by one son and one daughter (25). The next important combination of children appears to be two sons and two daughters (17 percent). About 6 percent of the couples indicated a preference for three sons and one daughter, while another 6 percent preferred two sons only. The remaining 12 percent of the currently married couples perceived other than the above, as best combination of children. Out of this 12 percent, only 6 percent of the respondents preferred more daughters than sons (wherein 5 percent wanted two daughters and one son) while others reported to be in favour of more sons than daughters (Khan and Prasad, 1983). The twelve stopping rules framed are :

Couples stop reproduction as soon as they give birth to:

Rule 1 : two children $(s = 2)$
Rule 2 : three children $(s = 3)$
Rule 3 : four children $(s = 4)$

feulc 4 : one son and one daughter $(b = I, g = I)$
Rule 5 : two sons $(b = 2)$
Rule 6 : one son and two daughters $(b = 1, g = 2\}$
Rule 7 : two sons and one daughter $(b = 2, g — 1)$
Rule 8 : two sons and two daughters $b = 2, g = 2)$
Rule 9 : three sons and one daughter $(b = 3, g = 1)$
Rule 10 : one son and one daughter or three children $(b = 1$ *and* $g = 1$, or $s = 3)$
Rule 11 : two sons and one daughter or four children $(b = 2$, and $g = 1$, or $s = 4)$
Rule 12 : two sons or three children $(b = 2$ or $s = 3)$

It is seen that Rules (4) to (9) are meant for those couples who wish to continue reproduction until desired minimum number of children by sex is achieved. Rules (1) to (3) are framed without any allowance for sex preference. The remaining three stopping rules (Nos. 10 to 12) regarding sex preference, are framed considering that it may be unrealistic to assume that couples will go on indefinitely having children until they achieve the desired minimum number of children by each sex.

Using the results given in the previous section the expected fertility trend for fulfilling the desire of couples in the above cases as well as for the control set have been computed corresponding to h = 1.5 and 1.75 years and " — .384 and .612. These values have been chosen arbitrarily but are consistent with the empirical estimate for Indian women. For example, the values of A (nine months of gestation plus nine to eleven months of postpartum amenorrhea) are based on the results of Potter *et al.* (1965), Saxena (1966) and Venkatacharya (1970). The estimates of ae are based on all India age specific fertility rates, and are also consistent with empirical estimates obtained elsewhere (Pathak and Sexena, 1979). The value of p is assumed to be .512 as mentioned earlier. Further the empirical value of (1—ßx) corresponding to ages 19, 24, 29, 34, 39 and 44 (0.0542,0.0284, 0.0949,0.1362, 0.3688 and 0.5823 respectively) is taken to derive single year value by fitting a 2nd degree polynomial curve from the results obtained by Yadava *et al.* (1982). The results are summarised in Tables 1-3.

Table 1 shows the Total Marital Fertility Rate (TMFR) for the control set and for the experimental set (under different sex preferences and stopping rules) during the period 1981-96. It is in fact shown for a year (1986) as the fertility rates for the period 1981-96 remain more or less stable under control set and under each stopping rule. Table 1 also shows that the proposed model is sensitive enough to indicate the variation in the level of fertility between lets of values of the parmeters TC and h under each stopping rule.

The likely impact of sex preference on current fertility is clearly evident when TMFRs under different stopping rules are compared (see Table 1). For a

TABLE 1—PROBABILITY (04) OF NOT ACHIEVING THE DESIRED NUMBER OF CHILDREN OF EACH SEX (*b* BOYS AND *g* GIRLS) BY PARITY m.

(*p* = .512.X = 0,e_a =0, $\alpha = 0$)

Desirtd Number of Children			*Parity (m)*							
Boys (b)	*Girls (g)*	*Total (b+g)*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*$*
1	0	I	.488	.238	.116	.057	.02$	.014	.077	.003
0	1	1	.512	.262	.134	.069	.035	.018	-009	.005
2	0	2		.738	.482	.295	.173	.099	.055	.030
0	2	2		.762	.518	.331	.203	.121	.071	.041
1	1	2		.500	.250	-125	.063	.032	.012	.008
3	0	3			.866	.669	.478	.322	.207	.129
0	3	3			.884	.705	.523	.367	.247	.161
2	1	3			.616	.363	.208	.117	.064	.035
1	2	3			.634	.387	.231	.135	.077	.044
4	0	4				.931	.797	.634	.474	.337
0	4	4				.943	.827	.679	.526	.390
3	1	4				.738	.513	.340	.217	.134
1	3	4				.762	.550	.380	.253	.164
2	2	4				.625	.376	.220	.126	.071

given set of values of the parameters TC and *h*, the lowest total fertility would be achieved if there was no sex preference. This holds true considering the variation in fertility under a given size of family (number of total children desired). The next lowest TMFR appears when the preference is for equal numbers of children by each sex. When the number of sons desired is greater than the number of daughters desired under a given size, TMFR is greater than that in the case where the preference is for equal numbers of children in each sex. The maximum is obviously reached when the desired minimum family consists of one sex only (all combinations are not shown in Table 1). In other

Words, it may be said that total fertility increases with increasing preference for one sex over the other The findings seem to be basically consistent with those of other related studies where the variation in family size under different rules adopted by the parents regarding the sex composition of their children, are examined through probability models (Sheps, 1963; Pathak, 1973).

In order to understand the implications of allowing couples to attain the desired family size and/or its sex composition on the couple's total fertility, TMFRs obtained under different stopping rules are compared with that of control set where fertility rates are derived without an allowance for a stopping point. It is evident from Table 1 that the current level of fertility can be reduced substantially even if all couples are allowed to have at least one son and one daughter (Rule 4) or two sons and one daughter (Rule 7). For example, in 1986 the expected total fertility rate for attaining the desired sex composition under rules 4 and 7 is found to be in the range of 2.63-2.75 and 3.63-3.96 respectively, while it is in the range of 5.02-6,97 (depending on the values of " and *h)* under the control set. It can, however, be seen from Table 1 that the TMFR for attaining the desired sex composition is higher under Rule 9 when compared with the same in the rest of the eleven hypothetical cases illustrated here. In this case, greater preference for size and sex (boys) is shown. For Rules 4 and 8, where a couple gives equal sex preference in case of two and four children, ihe TMFR is still lower than that obtained under Rules 5 and 9 respectively. Under Rules 10 and 12 a couple may stop reproduction even after two or three children provided the desired sex composition in the family is achieved, and hence the TMFR in case of Rules 10 and 12 is less than the same for getting three or four children under Rules 2 and 3 respectively. It is only for Rule 1, that the desire for a girl is not shown and a couple does not stop reproduction until two sons are born. The total fertility rate under Rule 5 is, therefore, almost equal to that of getting between three and four children, indicating that an extremely strong preference for sex can lead to a very high total fertility rate.

Table 2 shows the age Specific Marital Fertility Rate (ASMFR) for the year 1986 corresponding to the control set and experimental set. This is presented corresponding to " = .384 and *h* = 1.5 years for illustration. The pattern of ASMFR for any other year within 1981-96 is quite close to that of 1986, for the control set and for each of the stopping rules under experimental set, for a given set of values of the parameters TT and *h.* The impact of adopting rules on ASMFR is clearly evident, especially in the later age groups. It can be noticed that all the ASMFRs under the experimental set are lower than or equal to those of the control set for any age group. The ASMFRs for the later age groups, under the experimental set, are much lower than the corresponding ASMFR of control set. Thus the greater reductions in annual fertility (in terms of TMFR or GMFR) is obtained because of reduction in fertility in the middle and older age groups.

TABLE 2-AGE SPECIFIC MARITAL FERTILITY RATE (ASMFR) FOR THE CONTROL SET AND FOR THE DIFFERENT STOPPING RULES UNDER THE EXPERIMENTAL SET, 1986 (n= .384, *h* = 1.50 YEARS)

	ASMFR (Birthi per 1000 Married Women} Under Age Group					
	15-19	*20-24*	25-29	*30-34*	*35-39*	*40-44*
Control Set	142.47	223.41	230.83	209.88	167.86	106.61
Experimental Set						
Rule 1	140.68	157.40	68.04	18.34	3.69	0.54
Rule 2	142.47	210.25	143.32	57.16	15.31	2.82
Rule 3	142.47	222.5!	201.51	115.21	42.00	9.89
Rule 4	141.59	187.02	123.60	61-15	24.91	7.94
Rule 5	142.01	202.58	159.59	96.77	47.59	17.86
Rule 6	142.47	218.37	190.70	122,79	62.13	23.72
Rule 7	142.47	218.12	188.96	119.62	59.06	21.90
Rule *8*	142.47	223.07	218.43	161.87	90-46	36.54
Rule 9	142.47	223.16	221.85	173.27	105.32	46.86
Rule 10	141.59	183.S5	105.69	37.75	9.52	1.68
Rule 11	142.47	217.80	179.18	92.93	31.75	7.20
Rule 12	142.01	196.40	123.57	46.98	12.27	2.22

We have examined in detail birth rates for the control set and those of stopping rules 1 to!2 for selected years of the period 1981-96. The interpretation of the results calls for some caution. Here the birth rates are obtained by using birth probability matrices estimated on the basis of marriage cohorts. The interpretation of the results in Table 3 is more or less similar to that of Table 1. The differences between the birth rates for any of the 12 stopping rules and corresponding birth rates for the control set, indicate the implications of allowing couples to attain desired family size and/or composition, on the national birth rate From Table 3, it can be seen that the birth rates, based on a given set of values of the parameters " and *h,* for the period 1981-96 are more or less stable, except for a tendency to increase slightly in the later years (not shown for all the years in Table 3). This is due to the interaction between the changing age structure of the population and fertility rates.

The impact of sex preferences on current fertility is also clearly evident from Table 3. For a given family size, the lowest birth rate would be achieved if there were no sex preferences. It is also evident from Table 3 that the

TABLE 3—CRUDE BIRTH RATE FOR THE CONTROL SET AND FOR THE DIFFERENT STOPPING RULES UNDER THE EXPERIMENTAL SET, 1981-1996,

Year	*Fecundability* n	*Rest period h (in years)*	*Control Set*	*Crude Birth Rate (Births per 1000 population) under Experimental Set*											
				Rule I	*Rule 2*	*Rate 3*	*Rule 4*	*Rule 5*	*Rule 6*	*Ride 7*	*Rule 8*	*Rule 9*	*Rule 10*	*Rule 11*	*Rule 12*
1981	-384	1.50	32.04	11.76	17.90	22.97	16.84	20.54	23.48	23.20	26.76	27.82	14.85	21.03	16.30
		1.75	30.32	11.80	17.86	22.71	16.70	20.21	23.02	22.77	2607	26.96	14.84	20.84	16.28
	.612	1.50	41.59	11.23	17.94	24-11	17.27	22.02	25.62	25.18	30.16	32.06	14.60	21.75	16.19
		1.75	38.78	11.33	18.01	2402	17.22	21.76	25.24	24.84	29.51	31.17	14.68	21.72	16.27
1986	.384	1.50	31.10	11-58	17.21	22.07	16.34	19.87	22.64	22.35	25.84	26.89	14.41	20.21	15.74
		1.75	29.45	11.59	17.17	21.86	16.20	19.56	22.22	21.97	25.21	26.10	14.39	20.07	15.71
	.612	1.50	40.41	11.47	17.39	23.12	16.98	21.41	24.72	24.29	29.05	30.93	14.44	20.93	15.85
		1.75	37.71	11.48	17.41	23.05	16.88	21.14	24.36	23.97	28.48	30.12	14.46	20.89	15.86
1991	.384	1.50	31.11	12.43	17.91	22.54	16.98	20.34	22.98	22-72	25.98	27.00	15.18	20.68	16.4S
		1.75	29.47	12.42	17.78	22.09	16.80	19.93	22.51	22.28	25.31	26.18	15.11	20.44	16.38
	.612	1.50	40.43	12.52	18.69	24.00	17.96	22.23	25.48	25.08	29.50	31.27	15.62	21.97	17.08
		1,75	37.73	12.54	18.60	23.73	17-81	21.88	25.00	24.64	28.80	30.37	15.58	21.76	17.02
1996	.384	1.50	31.42	12.85	18.63	23.17	17.53	20.91	23.62	23.37	26.56	27.51	15.75	21.43	17.12
		1.75	29.76	12.84	18.50	22.81	17.34	20.53	21.11	2288	25.82	26.64	15.68	21.16	17.03
	.612	1.50	40.83	1282	19.40	25.11	18.54	22.95	26.34	25.95	30.44	32.13	16.13	22.93	17.69
		1.75	38.10	12.85	19.35	24.83	18.39	22.58	25.83	25.47	29.66	31.13	16.12	22.73	17.66

birth rate in India could be greatly reduced by an effective campaign of limiting family size to three or less. For example, in 1986, the birth rate of 31.1 per 1000 population observed under the control set (corresponding to n = .384 and A=1.5) which is quite close to the present level of birth rate in India, reduces by 62.8 percent under Rule 1 (2 children) and 44.7 percent under Rule 2 (3 children). Even if couples are allowed to have one son and one daughter (Rule 4) the birth rate of 31.10 declines by 47.5 percent, while the corresponding reduction is 28.1 percent under Rule 7 (2 sons and 1 daughter). Under Rule 7, a greater preference for sons is shown, which is an unrealistic assumption in the sense that families will indefinitely go on having children until they achieve the desired minimum number of each sex. Therefore, if each couple is allowed to have two sons and one daughter subject to a maximum of four children (Rule 11), the expected reduction in the birth rate is still more (35.0 percent) than that obtained under Rule 7.

Summary and Conclusions

Various probability models have been developed to show that if the sex bias is stable, the expected family size will increase with increasing preference for one sex over the other. Even when couples wish to have at least one son or at least one son and one daughter and keep having children until they attain this desired composition, the family size is shown to be higher than it would be otherwise. However, the effect of increasing preference for one sex over the other on the current fertility indices is not clearly known. In this regard, it would be interesting to know the implication of allowing couples, to attain the desired family size and composition for certain current fertility indices. This paper proposes a new mathematical model which may help answer ths related queries at the macro-level. For illustration, the model is applied to Indian data. The results regarding the effect of sex preference on fertility are basically consistent with the findings of other related studies discussed earlier, although the present model examines the effects on current fertility measures. The expected total fertility rate or the birth rate of the population increases with increasing preference for one sex over the other. Even if couples wish to have one son and one daughter and continue to have children until they achieve their desired composition, the total fertility rate or the birth rate of the population would always be higher than it would be if they stop at two children irrespective of the sex. Nevertheless, the results further reveal that the present birth rate in India could be reduced by almost half (48 percent) even if the couples are allowed to have one son and one daughter.

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APPENDIX-I

The Details of the Proofs of the Formulae Presented in the Text for the Estimation of Birth Probability Matrix

In the experimental group a couple is assumed to stop reproduction as soon as it achieves the desired level of b sons and g daughters ($b, g \succeq 0$). Let D_m be the event that the desired level of b and g is not achieved by the couple at the mth parity or before. In other words, for $m > b + g$,

$$D_m = (\text{Among the } m \text{ children the number of boys is atmost } b-1) \cup (\text{Among the } m \text{ children the number of girls is atmost } g-1)$$

Therefore, the probability, $P(D_m)$, that desired level is not reached at the mth parity or before, denoted by $Q_m(b, g)$, is given by

$$Q_m (b, g = \begin{cases} 1 & \text{if } m < b+g \\ \sum_{r=0}^{b-1} \binom{m}{r} p^r q^{m-r} + \sum_{r=0}^{g-1} \binom{m}{r} q^r p^{m-r} & \text{if } m > b+g \end{cases}$$

where p is the probability that a live birth is male.

While presenting the formulae for age at marriage and age specific birth probabilities in the main text, the various assumptions behind the model have already been discussed. For the convenience of computation, the probability of a woman giving birth to a child at a specific age is taken as the probability of a woman giving birth to a child in the corresponding year of marriage. (This is the case if the date of marriage and woman's birth date are almost close) It is further assumed that there is one to one correspondence between conception and live birth and the gestation period associated with it is constant and equal to 9 months or 3/4 year. The probability of occurrence of a birth in a given year depends on the timing of conception.

Let us consider time sequence of occurrence of births to a woman during the reproductive period since her marriage. Let '0' coincide with the date of marriage, the points 1, 2, . . . , k denote completion time points of the 1st, 2nd, . . . , kth year of marriage and $t_k = k - 1 + \frac{1}{4} = k - \frac{3}{4}$.

1st year	2nd year	3rd year	$(k-1)$th year	kth year
$0 \; t_1$	$1 \; t_2$	$2 \; t_3 \; 3$	$k-2, \; t_{k-1}$	$k-1 \; t_k \; k$

It may be noted that the interval $[t_{k-1}, t_k]$ denotes the conception time interval for a birth in the kth year of marriage.

Let W_k be the waiting time for the kth conception and h be the rest period (gestation + PPA) associated with a conception. The total waiting times for the 1st, 2nd, . . . , kth conception from the date of marriage are W_1, $W_1 + W_2 + h, \ldots, W_1 + W_2 + \ldots + W_k + (k-1)h$ respectively.

Let $C_1, C_2, \ldots, C_k$ denote the event that a birth takes place in the 1st, 2nd, . . . , kth year of marriage in the control group where a couple does not follow any stopping rule regarding preference for sex composition and/or size of the family. A birth takes place to a woman in kth year under the condition that she is not sterile in the relevant period. For convenience of notation, the conditioning event is ignored but to show that event is conditional, its probability is indicated by an asterisk. Having obtained conditional probabilities, unconditional probabilities can be obtained by multiplying them by the relevant probabilities of non-sterility. Then,

C_k = The event that a birth takes place in the kth year of marriage

= (A birth of order one takes place in the kth year of marriage) $\cup$
(A birth of order two takes place in the kth year of marriage) $\cup$

- -

- -

(A birth of order k takes place in the kth year of marriage)

= (A conception, leading to 1st order birth, takes place in the interval (t_{k-1}, t_k)) $\cup$
(A conception, leading to 2nd order birth, takes place in the interval (t_{k-1}, t_k)) $\cup$

- -

- -

(A conception, leading to kth order birth, takes place in the interval (t_{k-1}, t_k))

$$= [t_{k-1} < W_1 < t_k] \cup [t_{k-1} < W_1 + W_2 + h < t_k] \cup \ldots$$
$$\ldots [t_{k-1} < W_1 + W_2 + \ldots + W_{k-1} + (k-1)h + W_k < t_k]$$

$$= \bigcup_{m=1}^{k} [t_{k-1} < W_1 + W_2 + \ldots + W_{m-1} + (m-1)h + W_m < t_k]$$

where $W_0 \equiv 0$. In other words, the event C_k can be written as

$$C_k = \bigcup_{m=1}^{k} C_{k,m}$$

where

$$C_{k,m} = [t_{k-1} < W_1 + W_2 + \ldots + W_{m-1} + (m-1)h + W_m < t_k]$$

and $C_{k,1} \ldots C_{k,k}$ are mutually exclusive events. Since $W_1, W_2, \ldots, W_k$ are independent identically distributed exponential random variables with parameter π, $W_1 + W_2 + \ldots + W_k$ follows gamma distribution, given by its probability density

$$f(x; k, \pi) = \begin{cases} \dfrac{\pi^k}{\sqrt{k}}\; e^{-\pi x}\, x^{k-1} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

where the parameters k and π are both positive. Thus,

$$\begin{aligned} P^*(C_k) &= \sum_{m=1}^{k} Pr \cdot (C_{k,\,m}) \\ &= \sum_{m=1}^{k} Pr \cdot [t_{k-1} \leqslant W_1 + W_2 + \ldots + W_{m-1} + (m-1)\,h + W_m < t_k] \\ &= \sum_{m=1}^{k} Pr \cdot [t_{k-1} - (m-1)\,h < Z_m < t_k - (m-1)\,h] \end{aligned}$$

where, $Z_m = W_1 + W_2 + \ldots + W_m$

$$\begin{aligned} P^*(C_k) &= Pr \cdot [Z_m > t_{k-1} - (m-1)\,h] - Pr \cdot [Z_m > t_k - (m-1)\,h] \\ &= Pr \cdot [Z_m > r] - Pr \cdot [Z_m > r + 1], \end{aligned}$$

where,

$$r = t_{k-1} - (m-1)\,h.$$

For $r > 0$ the term $Pr \cdot [Z_m > r]$ is evaluated as follows :

$$\begin{aligned} Pr \cdot [Z_m > r] &= \int_r^a f(x; m, \pi)\, dx \\ &= \int_r^a \frac{\pi^m}{\sqrt{m}}\, x^{m-1}\, e^{-\pi x}\, dx \\ &= \frac{\pi^m}{\sqrt{m}} \left[-\frac{1}{\pi}\, x^{m-1}\, e^{-\pi x} \right]_r^a \\ &\quad - \frac{\pi^m}{\sqrt{m}} \int_r^a \left[-\frac{1}{\pi}\, e^{-\pi x} \right] (m-1)\, x^{m-2}\, dx \\ &= \frac{\pi^{m-1}}{(m-1)\,!}\, r^{m-1}\, e^{-\pi r} + \frac{\pi^{m-1}}{\sqrt{m-1}} \int_{rx}^{a} (m-1) - 1\; e^{-\pi x}\, dx \\ &= \frac{e^{-\pi r}\, (\pi r)^{m-1}}{(m-1)\,!} + Pr \cdot [Z_{m-1} > r] \\ &= \sum_{n=0}^{m-1} \frac{e^{-\pi r}\, (\pi r)^n}{n\,!}, \end{aligned}$$

the cumulative probability upto $(m - 1)$ of the Poisson distribution with mean πr.

Similarly,

$$Pr \cdot [Z_m > r + 1] = \sum_{n=0}^{m=1} \frac{e^{-\pi(r+1)}}{n!} \{\pi(r + 1)\}^n$$

Therefore, the probability of a woman giving birth in the kth year of marriage in the control group is given by

$$P^*(C_k) = \sum_{m=1}^{k} Pr\,[r < Z_m < r + 1]$$

where,

$$Pr \cdot [r < Z_m < r + 1] = \begin{cases} \sum_{n=0}^{m-1} \frac{e^{-\pi r}(\pi r)^n}{n!} - \sum_{n=0}^{m-1} \frac{e^{-\pi(r+1)}\{\pi(r + 1)\}^n}{n!} & \text{if } r > 0 \\ 1 - \sum_{n=0}^{m-1} \frac{e^{-\pi(r+1)}\{\pi(r + 1)\}^n}{n!} & \text{if } r \leqslant 0 < r + 1 \\ 0 & \text{if } r + 1 \leqslant 0 \end{cases}$$

and

$$r = t_{k-1} - (m - 1)\,h, \quad \text{and} \quad t_k = k - \frac{3}{4}$$

It should be noted that k does not exceed 45 — y, where y is the age at marriage of a woman. In fact, for $k > 45—y$, $P^*(Ck) = 0$. It may be recalled that probability of a woman giving birth to two children in the same year is assumed to be zero. Therefore, the maximum number of children in the fcth year of marriage is k. This assumption, however, can be relaxed easily. In that case, the maximum number of children by the kth year of marriage is K/G.

Let $Ek(S(b, g, s))$ denote the event that a birth takes place in the kth year of marriage in the experimental group where a couple follows scheme $S(b, g, s)$ to satisfy their desired number of sons (b) and daughters (g) and/or total number of children (j).

The derivation of $P^*(Ek)$ (and P'(Ek)), the conditional (and unconditional) probability that a birth takes place in the kth year of marriage of a woman in the experimental group, is similar to that of P*(Ck) (and $P(C_k)$) in the control group.

In order to find $P^*(E_k(S(b, g, s)))$ it is noted that

$$E_1 = C_1 = [t_0 < W_1 < t_1], \text{ where } t_0 = 0$$

and for $k \geqslant 2$

$$E_k = [t_{k-1} < W_1 < t_k] \cup \left[\bigcup_{m=0}^{k-1} \{t_{k-1} < W_1 + W_2 + \ldots + W_{m+1} + mh < t_k\} \cap D_m \right]$$

The event D_m and probabilities of its occurrence have already been discussed in the preceding section. It is now possible to obtain $P^*(E_1)$ and $P^*(E_k)$ on the same lines **as** $P^*(C_1)$ and $P^*(C_k)$ are derived.

Substituting $t_h = k - 3/4$ the expression for $P^*(C_k)$ and $P^*(E_k)$ given in the text, can be obtained. Further k does not exceed $45-y$, where y is the age at marriage of woman.

APPENDIX -II

Projection of Currently Married Women

To project the currently married women (aged 15-44), the following assumptions are made :

1. Marriage is assumed to be universal and no remarriage is considered. The age specific marriage probability (m_t) is assumed to correspond to the pattern shown in Appendix Table 1.

APPENDIX TABLE 1-PROBABILITY OE A WOMAN MARRYING AT AGE *x*

(in years)

x	m_x	x	m_a	
15	.4650	26	.0030	
16	.1100	27	.0025	
17	.0995	28	.0025	
18	.0810	29	,0025	
19	.0805	30	.0005	Mean = 16.965
20	.0640	31	.0005	Variance = 6.875
21	.0505	32	.0005	
22	-0145	33	.0005	
23	.0097	34	.0003	
24	.0073	35	.0002	
25	.0050			

SOURCE : Computed from Govt. of India, 1983. *Report & Tables.* Based on 5 percent Sample Data. Census of India, 1981 : Series 1, part II Special. Office of the Registrar General, New Delhi, India, pp. 366-368.

2. The level of mortality for males and females is assumed to correspond to the e_{00} shown in Appendix Table 2.
3. To obtain the joint survival ratio of a woman in married state, it is assumed that age differential between husband and wife is 5 years. Making use of e_{00} for the concerned period (see assumption 2), the single year survival ratios for males and females (Smx and *Sfx)* are first obtained from the results of a paper by Sinha (1972) which provides complete life tables based on Coale and Demeny's Model (West) Life Tables.

APPENDIX TABLE 2—EXPECTATION OF LIFE AT BIRTH (FEMALES FOR THE PERIOD 1981-96

Period	e_0^0	
	Male	Female
1981 -86	5S.6	56.2
1986-91	58.1	58.7
1991-96	60.6	61.2

SOURCE : Govt-of India, 1984. Population Projections for India 1981-2001. Census of India, 1981. Series-I, India. Office of the Registrar General, New Delhi, p. 9.

Appropriate joint survival ratio (S_{mfx}) is obtained as $S_{mfx} = S_{fx} \times S_{mx+5}$ ($x = 15, 16, \ldots, 44$).

4. The ratio of newly married women in a year to the currently married women of the age group 15-44 in the preceding year, is assumed to be constant throughout the projection period. This ratio is estimated to be around 5.5 percent from the census data of 1971 and 1981. This estimate is based on the method followed by Venkatacharya (1972, p. 357).
5. The proportion m_f of women marrying at a particular age.y in a year to all women that got married during the same year, is assumed to be constant throughout the projection period. Under this assumption, in the year J, $m_{jy} = P$ (a newly wed woman in the year J is of age y)
$$= m_y.$$

The 1981 census has revealed that the population of India is 685.185 million as on 1st March 1981. Further, the total number of currently married females in the age group 15-44 is reported to be about 115.776 million. The quinquennial age distribution of the currently married females based on five percent sample data (Govt. of India, 1983) is used to derive single year age distribution. To project the 1981 single year currently married females (in the age group 15-44) at each future year, the following procedure is adopted.

Let W_{JX} = the number of currently married women aged x in the Jth year (J= 1981, 1982, ... 1996)

W_J = the total number of currently married women in the age group 15-44 in the /th year

$$\left(W^J = \sum_{x=15}^{44} W_x^J \right)$$

and E_{JX} — the number of new entrants through marriage into the currently married group at age x in the /th year.

In view of assumptions 4 and 5 mentioned earlier, we haVe

$$E_x^J = (0.055)\, W^{J-1}\, m_x$$

Therefore, the number of currently married women aged x in the Jth year is given by

$$W_x^J = W_{x-1}^{J-1}\, S_{x-1}^{mf} + E_x^J \, .$$

This procedure is repeated from 1981 onward to obtain age specific currently married women for each of the years during 1981-96.